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**School of Electronics**
**CURRICULUM: IIITUGECE20**
**END SEMESTER EXAMINATION**
**25-04-2022**

| **Degree** | B. Tech. | **Branch** | ECE |
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| **Semester** | I | | |
| **Subject Code & Name** | MAC 121: Mathematics-I | | |
| **Time: 90 Minutes** | **Answer All Questions** | | **Maximum: 50 Marks** |

| Sl. No. | Question | Marks |
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| 1.a | Find the Eigen values and corresponding Eigen vectors of the following matrix. $A=\begin{pmatrix}-1 & 2 & -2\\ 1 & 2 & 1\\ -1 & -1 & 0\end{pmatrix}$ | (2) |
| 1.b | Find model matrix and spectral matrix of A. | (2) |
| 1.c | Prove that the square matrix A and $P^{-1}AP$ have the same Eigen values, where P is non-singular matrix of the same order as A. | (2) |
| 1.d | Verify Caley-Hamilton theorem for the following matrix $A=\begin{pmatrix}-1 & 2 & -2\\ 1 & 2 & 1\\ -1 & -1 & 0\end{pmatrix}$. | (4) |
| 2.a | Examine the convergence of the series $1+\frac{1}{2}+\frac{1.3}{2.4}+\frac{1.3.5}{2.4.6}+....$ | (2) |
| 2.b | Discuss the convergence of the series $x+\frac{2^2x^2}{2!}+\frac{3^3x^3}{3!}+\frac{4^4x^4}{4!}+....$ | (2) |
| 2.c | Prove that $1+\frac{e^{-2x}}{2^2-1}-\frac{e^{-4x}}{4^2-1}+\frac{e^{-6x}}{6^2-1}-...$ is uniformly convergent for $x\geq 0$. | (2) |
| 2.d | Show that the series $\sum_{1}^{\infty}\frac{1}{n^2+1}$ is convergent | (4) |

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| 3.a | Examine the function $y = 2\sin x + \cos 2x$ for maximum and minimum in the interval $[0, 2\pi]$. | (2) |
| 3.b | Inspect the area of the region enclosed by the parabolas $y = x^2$ and $y = 2x - x^2$. | (2) |
| 3.c | Utilizing $\varepsilon - \delta$ definition, show that $\lim_{x \to 3}(4x-5) = 7$. | (2) |
| 3.d | Inspect the function $f(x) = x^3 - x$ to satisfy the hypothesis of the Lagrange's mean value theorem in [0, 2]. Then find all numbers 'c' which satisfy the conclusion of the Lagrange's mean value theorem. | (4) |
| 4.a | Let V be the set of all real valued continuous functions f on $[a, b]$ such that $\int_a^b f(x)\,dx = 2$ with usual addition and scalar multiplication for functions. Identify whether V is a vector space. | (2) |
| 4.b | Making use of vector space, write the definition of a subspace. | (2) |
| 4.c | Let V be the set of all $3 \times 1$ real matrices with usual matrix addition and scalar multiplication. Also let W consisting of all $3 \times 1$ real matrices of the form $\begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$. Identify whether W is subspace of V. | (2) |
| 4.d | Choose the following set of vectors {(1, 1, 1), (0, 1, 1), (1, 0, 1)} and discuss whether it forms a basis in $\mathbb{R}^3$. | (4) |
| 5 | Let $T : \mathbb{R}^3 \to \mathbb{R}^2$ be a transformation given by $T(x, y, z) = (x + y,\ x - z)$ | |
| 5.a | Show that T is a Linear transformation | (2) |
| 5.b | Demonstrate the matrix representation of T with respect to the bases $X = \{(1,\ 0,\ 1),\ (1,\ 1,\ 0),\ (0,\ 1,\ 1)\}$ in $\mathbb{R}^3$ and $Y = \{(1,\ 3),\ (2,\ 5)\}$ in $\mathbb{R}^2$ | (2) |
| 5.c | Demonstrate the inverse of the matrix obtained in (5.b), if it is invertible. | (2) |
| 5.d | Let $T : M_{2\times2} \to \mathbb{R}^3$ be a linear transformation defined by $T\left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, T\left[\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}, T\left[\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix},$ $T\left[\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right] = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$. Demonstrate $T\left[\begin{pmatrix} 4 & 5 \\ 3 & 8 \end{pmatrix}\right]$. Here $M_{2\times2}$ denotes the set of all $2 \times 2$ matrices. | (4) |